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A support theorem for a wave equation

by

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Abstract

It is well known that the fundamental solution to the classical wave equation $\Delta u(x,t) - \partial_{tt}u(x,t) = 0$ is supported on the light cone $\{(x,t) \in \mathbb{R}^n \times \mathbb{R} : \ ||x|| = |t| \}$ if and only if the dimension n is odd and ≥ 3. Because we are living in a 3-dimensional world we can hear each other clearly; One has a pure propagator without residual waves. In this thesis we consider the wave equation

$$2\|x\|\Delta_k u_k(x,t) - \partial_{tt}u_k(x,t) = 0, \qquad (x,t) \in \mathbb{R}^n \times \mathbb{R},$$

where Δ_k is a second order differential and difference operator. First, we prove the existence and the uniqueness of the solution $u_k(x,t)$. Second, we search for the condition on the parameter k and the dimension n for the fundamental solution to be supported on the light cone

$$\{(x,t) \in \mathbb{R}^n \times \mathbb{R}: \ \sqrt{2||x||} = |t|\}$$

Our approach is based heavily on the representation theory of the Lie algebra $\mathfrak{sl}(2,\mathbb{R})$ where we construct a new representation ω_k of $\mathfrak{sl}(2,\mathbb{R})$ acting on the Schwartz space $S(\mathbb{R}^n)$. Finally, we prove that ω_k lifts to give raise to a unitary representation of a simply connected Lie group with Lie algebra $\mathfrak{sl}(2,\mathbb{R})$.

Keywords: Dunkl operators, wave equation, conservation of total energy, generalized Fourier transform, convolution structure, Huygens' principle, the Lie algebra $\mathfrak{sl}(2,\mathbb{R})$, representation theory of Lie algebras, integrability of infinitisimal representations.