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Universal Constraints of Kleinian Groups and Hyperbolic Geometry

by

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Date & Venue

10:00 AM

Monday, 29 June 2020

https://eu.bbcollab.com/guest/f751768f7c9a4a4d81d42c23c6921939

Abstract

Recent advances in geometry have shown the wide application of hyperbolic geometry not only in mathematics but also real world applications. As in two dimensions, it is now clear that most three-dimensional objects (configuration spaces and manifolds) are modelled on hyperbolic geometry. This viewpoint explains a great many things from large-scale cosmological phenomena, such as the shape of the universe, right down to the symmetries of groups and geometric objects and various physical theories. Kleinian groups are basically discrete groups of isometries associated with tessellations of hyperbolic space. They form the fundamental groups of hyperbolic manifolds. Over the last few decades, the theory of Kleinian groups has flourished because of its intimate connections with low-dimensional topology and geometry, especially with 3-manifold theory.

In particular, we seek generalizations of known universal constraints for Fuchsian groups - discrete subgroups of isometries of hyperbolic plane. These generalizations will underpin a new understanding of the geometry and topology of hyperbolic 3-manifolds and their associated singular spaces, hyperbolic 3-orbifolds. The universal constraints for Kleinian groups we will seek in part arise from a novel description of the moduli spaces of discrete groups. This approach was successfully used to complete the solution to Siegel's famous problem on hyperbolic lattices, and offers further substantive advances to address the quite complicated analytic and topological properties of hyperbolic orbifolds. Our novel approach is to use a fundamental result concerning spaces of finitely generated Kleinian groups: they are closed in the topology of algebraic convergence. Indeed, this is also true in higher dimensions when fairly minor additional (and necessary) conditions are imposed – for instance giving a uniform bound on the torsion in a sequence, or asking that the limit set be in geometric position. In fact, this property (which is basically a consequence of the existence of Zassenhaus neighbourhoods for semi-simple Lie groups) holds more generally for groups of isometries of negatively curved metrics because of the Margulis-Gromov lemma.

We use new polynomial trace identities in the Lie group SL(2,C) to establish new universal constraints for Kleinian groups, and the geometry of associated three-manifolds, thereby advancing the solutions of important unsolved problems.

Keywords: *Kleinian groups*, *complex triple parameters, moduli spaces*, *universal constraints*, *hyperbolic geometry.*