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Entitled

STOCHASTIC DELAY DIFFERENTIAL EQUATIONS WITH APPLICATIONS IN ECOLOGY AND EPIDEMICS

by

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Date & Venue

11:00 AM

Thursday, 12 November 2020

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Abstract

Mathematical modeling with *delay differential equations* (DDEs) is widely used for analysis and predictions in various areas of life sciences, such as population dynamics, epi- demiology, immunology, physiology, and neural networks. The memory or time-delays, in these models, are related to the duration of certain hidden processes like the stages of the life cycle, the time between infection of a cell and the production of new viruses, the duration of the infectious period, the immune period, and so on. In *ordinary differential equations* (ODEs), the unknown state and its derivatives are evaluated at the same time instant. In DDEs, however, the evolution of the system at a certain time instant depends on the past history/memory. Introduction of such time-delays in a differential model significantly improves the dynamics of the model and enriches the complexity of the system.

Moreover, natural phenomena counter an environmental noise and usually do not fol- low deterministic laws strictly but oscillate randomly about some average values, so that the population density never attains a fixed value with the advancement of time. Accordingly, *stochastic delay differential equations* (SDDEs) models play a prominent role in many appli- cation areas including biology, epidemiology and population dynamics, mostly because they can offer a more sophisticated insight through physical phenomena than their deterministic counterparts do. The SDDEs can be regarded as a generalization of *stochastic differential equations* (SDEs) and DDEs.

This dissertation, consists of eight Chapters, is concerned with qualitative and quan- titative features of *deterministic* and *stochastic delay differential equations* with applications in ecology and epidemics. We investigate local and global stabilities of the steady states and Hopf bifurcations with respect of interesting parameters of such models. We study the im- pact of incorporating time-delays and random noise in such class of differential equations for different types of prey-predator systems and infectious diseases. Numerical simulations, using suitable and reliable numerical schemes, are provided to show the effectiveness of the obtained theoretical results.

Chapter I provides a brief overview about the topic and shows significance of the study. *Chapter II*, is devoted to investigate the qualitative behaviours (through local and global stability of the steady states) of DDEs with prey-predator systems in case of hunting cooperation on predators. *Chapter III* deals with the dynamics of DDEs, of multiple time- delays, of two-prey one-predator system, where the growth of both preys populations subject to Allee effects, with a direct competition between the two-prey species having a common predator. A Lyapunov functional is deducted to investigate the global stability of positive interior equilibrium. *Chapter IV*, studies the dynamics of stochastic DDEs for prey-predator system with hunting cooperation in predators. Existence and uniqueness of global positive solution and stochastically ultimate boundedness are investigated. Some sufficient condi- tions for persistence and extinction, using Lyapunov functional, are obtained. *Chapter V* is devoted to investigate Stochastic DDEs of three-species prey-predator system with cooper- ation among prey species. Sufficient conditions of existence and uniqueness of an ergodic stationary distribution of the positive solution to the model are established, by constructing a suitable Lyapunov functional. *Chapter VI* deals with stochastic epidemic SIRC model with time-delay for spread of COVID-19 among population. We deduce the basic reproduction number R_0^s for the stochastic model which is smaller than R_0 of the corresponding determin- istic model. Sufficient conditions that guarantee the existence of a unique ergodic stationary distribution, using the stochastic Lyapunov functional, and conditions for the extinction of the disease are obtained. In *Chapter VII*, we briefly discuss some numerical schemes for SDDEs. Convergence and consistency of such schemes are investigated.

Chapter VIII summaries our finding and future directions of research.

Our findings, theoretically and numerically, show that time-delays and random noise have a significant impact in the dynamics of ecological and biological systems. They also have an important role in ecological balance and environmental stability of living organisms. A small scale of white noise can promote the survival of population; While large noises can lead to extinction of the population, this would not happen in the deterministic systems without noises. Also, white noise plays an important part in controlling the spread of the disease; When the white noise is relatively large, the infectious diseases will become extinct; Re-infection and periodic outbreaks can also occur due to the time-delay in the transmission terms.

Keywords: Allee effect; Bifurcation; Brownian motion; Epidemic models; Lyapunov functionals; Prey-predator model; Sensitivity; SIRC; Stability; Stationary distribution; Stochastic perturbations; Time-delays