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Entitled
ON THE GENERALIZED HARDY-LITTLEWOOD MAXIMAL OPERATOR.
by
Namarig Hashim Elfadil Hassan
Faculty Advisor
Dr. Salem Ben Said, Department of Mathematical Sciences
College of Science
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Abstract

In this report we introduce and then study a maximal operator that generalizes the classical one introduced by Hardy and Littlewood in the rank one case. More precisely, for $k \geq 0$ and an integer n,

$$M_{k,n}f(x) = \sup_{r>0} \frac{1}{\mu_{k,n}(]-r,r[)} \left| \int_{\mathbb{R}} f(y)\, \tau_x^{k,n}(\chi_r; y)\, d\mu_{k,n}(y) \right|,$$

where the measure $\mu_{k,n}$ is given by $d\mu_{k,n}(y) = |y|^{2k+\frac{2}{n}-2}\, dy$, and $\tau_x^{k,n}$ is a certain translation operator. The main result is to prove the weak $(1,1)$ inequality and the strong (p,p) inequality for $M_{k,n}$,with $1 < p \leq \infty$. The approach uses geometric and analytic tools. One of the major technical obstacles is the lack of known properties of the translation operator $\tau_x^{k,n}$. The strategy is to introduce an uncentered maximal operator associated to intervals of type $I(x,r) =](max\{0, |x|^{\frac{1}{n}} - r^{\frac{1}{n}}\})^n, (|x|^{\frac{1}{n}} + r^{\frac{1}{n}})^n[$ which controls the maximal operator $M_{k,n}$. To do so, one needs to prove a Vitaly type covering lemma for the intervals $\{I(x_j, r_j)\}_j$, together with a sharp estimate for $\mu_{k,n}\left(I(x_j, y_j)\right)$. The main result generalizes the case $n = 1$ proved by Rösler, and the case $n = 2$ proved by Ben Said and Deleaval.

Keywords: Hardy-Littlewood maximal operator. Generalized Fourier transform. Vitaly type lemma. Strong and Weak type inequalities. Convolution structure. Translation operator.